

Important Concepts . . .

Preview Review



Mathematics

Grade 6

**W1 - Lesson 3: Improper Fractions
and Mixed Numbers**

Important Concepts of Grade 6 Mathematics

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Materials Required: A textbook is not needed. This is a stand-alone course.

Mathematics Grade 6

Version 5

Preview/Review W1 - Lesson 3

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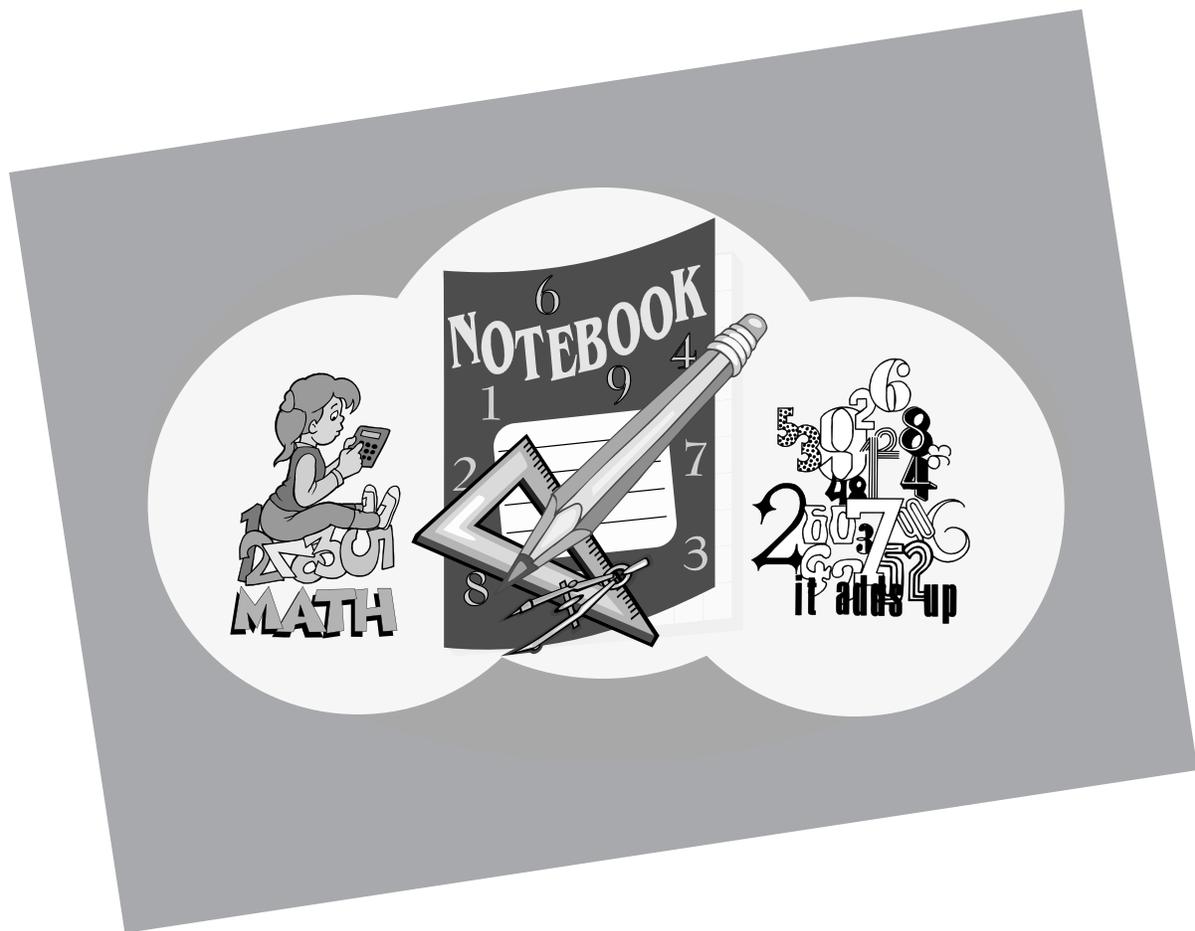
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Preview/Review Concepts for Grade Six Mathematics



*W1 - Lesson 3:
Improper Fractions and
Mixed Numbers*

OBJECTIVES

By the end of this lesson, you should

- change proper and improper fractions into decimals and fractions
- change decimal fractions into proper and improper fractions
- write fractions and mixed numbers in both word and numeral forms
- create equivalent fractions by multiplying and dividing the numerators and denominators by whole numbers
- arrange groups of numbers according to size
- change mixed numbers into improper fractions
- change fractions into mixed numbers

GLOSSARY

common denominator - a denominator that is the same in two or more fractions

improper fraction - a fraction in which the numerator is greater than or equal to the denominator to produce a value greater than or equal to one

lowest term fraction - a fraction that is the smallest equivalent fraction possible

mixed number - a number that contains a whole number and a fraction written together to produce a value greater than the value of the whole number it contains

proper fraction - a fraction in which the numerator is less than the denominator to produce a value less than one

W1 - Lesson 3: Improper Fractions and Mixed Numbers

Welcome to W1 - Lesson 3! In this lesson you will continue to work with fractions in three topics:

- Mixed Numbers and Improper Fractions
- Changing Fractions to Decimals
- Comparing Proper Fractions, Improper Fractions, and Mixed Numbers

To review the concepts, give careful attention to each section and to each assignment.

Mixed Numbers and Improper Fractions

You likely use proper fractions often. You ate three-quarters of the pizza; you drank half the pop; you did a quarter of your homework. These are proper fractions. Each is less than one. The denominator tells the number of pieces the pizza is cut into, the numerator tells how many you ate!



A **mixed number** is a whole number that is written together with a fraction.

$$2 \frac{3}{4}, 5 \frac{7}{10}, 11 \frac{7}{25}, 8 \frac{11}{100}, \text{ and } 34 \frac{22}{1\ 000}$$

The value of the whole number and the value of the fraction together make the value of the mixed number.

An **improper fraction** can be a number with a numerator the same as the denominator. If you need 5 pieces of pie to make one complete pie and you have those 5 pieces, you can put them together to make one pie. Any number divided by itself equals 1.

$$\frac{5}{5} = 1 \quad \frac{5}{5} \div 5 = \frac{1}{1} = 1$$

Most improper fractions have numerators that are larger than the denominators.

Having a fraction with the numerator larger than the denominator is like having a container that is heaped so you cannot put the lid on. The value is more than one because the fraction has more parts than it needs to make

one. That is why $\frac{5}{4}$ fills a container that contains 4, and 1 is left over.

Improper fractions are also written as mixed numbers.

$$\frac{5}{4} = 1\frac{1}{4} \quad \frac{8}{3} = 2\frac{2}{3} \quad \frac{17}{5} = 3\frac{2}{5}$$



Questions

1. Determine if the following numerals are **proper fractions**, **improper fractions**, or **mixed numbers**. Write the correct answer in the space provided.

Examples: $3\frac{6}{10}$ = mixed number, $\frac{8}{7}$ = improper fraction, $\frac{7}{10}$ = proper fraction

a. $12\frac{4}{9}$ = _____

b. $16\frac{3}{7}$ = _____

c. $\frac{443}{1\ 000}$ = _____

d. $5\frac{2}{3}$ = _____

e. $\frac{9}{8}$ = _____

2. Rewrite the following numerals in words. Remember to use ***ths*** where needed, as in **tenths**, **hundredths**, and **thousandths**.

Example: $33\frac{5}{9}$ = thirty-three and five ninths

a. $\frac{83}{100}$ = _____

c. $\frac{46}{11}$ = _____

d. $\frac{78}{2} =$ _____

e. $38 \frac{8}{14} =$ _____

f. $\frac{3}{19} =$ _____

3. Rewrite the following numbers in numerals.

Example: seventy-seven and twenty-two hundredths = $77 \frac{22}{100}$

a. nine thirds = _____

b. seven twentieths = _____

c. two hundred seven and one quarter = _____

d. sixteen halves = _____

e. eight eighths = _____

f. nineteen and nineteen fiftieths = _____

4. Make three equivalent fractions for each of the following fractions by **multiplying**. Choose your own multipliers.

Example: $\frac{11}{5} = \frac{22}{10}, \frac{33}{15}, \frac{44}{20}$ The multipliers are 2, 3 and 4.

a. $\frac{7}{8} =$

c. $\frac{13}{13} =$

d. $\frac{6}{4} =$

e. $\frac{22}{10} =$

f. $\frac{44}{50} =$



5. Make three equivalent fractions by **dividing**. Choose your own divisors.

Example: $\frac{250}{100} = \frac{125}{50}, \frac{50}{20}, \frac{25}{4}$ The divisors are 2, 5 and 10.

a. $\frac{500}{100} =$ _____

b. $\frac{66}{66} =$ _____

c. $\frac{88}{120} =$ _____

d. $\frac{1\ 000}{800} =$ _____

e. $\frac{250}{350} =$ _____

f. $\frac{60}{30} =$ _____



Changing Fractions to Decimals

Earlier, you reviewed equivalent fractions.

You know that $\frac{1}{2}$ is equal to $\frac{2}{4}$ or $\frac{3}{6}$.

The fraction $\frac{2}{4}$ is not lowest-terms because both numerator and

denominator can be divided by 2. This changes $\frac{2}{4}$ to $\frac{1}{2}$, which is a **lowest-terms fraction** because we cannot divide any whole number into both the numerator and the denominator.

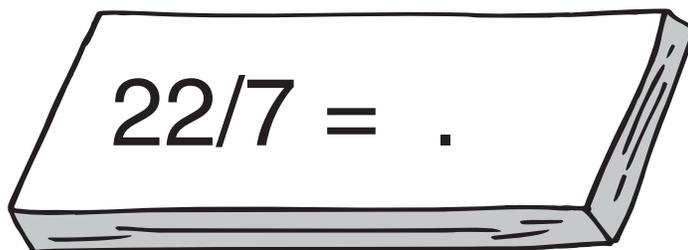
$\frac{3}{5}$ is a lowest-term fraction because both the numerator and denominator cannot be divided by any whole number.

$\frac{4}{10}$ is not lowest-terms because both 4 and 10 are divisible by 2.

When an equivalent fraction is made by dividing, the new lowest term fraction is $\frac{2}{5}$.

$\frac{12}{20}$ is not lowest-terms because both 12 and 20 are divisible by 2 and by 4. We can make two equivalent fractions that are lower.

$\frac{12}{20} = \frac{6}{10} = \frac{3}{5}$ The fraction $\frac{3}{5}$ is lowest-terms.



Another way to express a fraction is as a **decimal fraction**. If the denominator is 10, this is easy because of place values. Three tenths is written as $\frac{3}{10}$ or as 0.3. The numerator becomes the digit in the tenths

place value location. If $\frac{9}{10} = 0.9$, then $\frac{29}{100} = 0.29$

Examples:

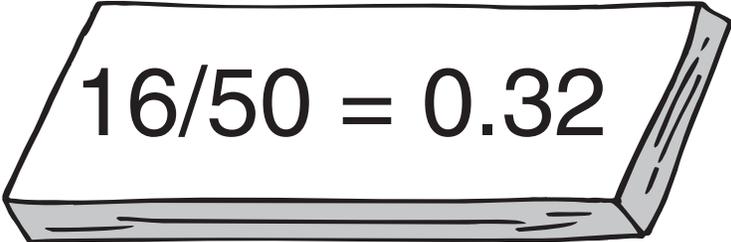
$$\frac{7}{10} = 0.7 \text{ (The 7 is in the tenths place value location.)}$$

$$\frac{44}{100} = 0.44 \text{ (There must be two numbers after the decimal to be in the hundredths place value location.)}$$

$$\frac{701}{1\ 000} = 0.701 \text{ (There must be three numbers after the decimal to be in the thousandths place value location.)}$$

$$\frac{8}{100} = 0.08 \text{ (There must be two numbers after the decimal to be in the hundredths place value location.)}$$

$$\frac{23}{1\ 000} = 0.023 \text{ (There must be three numbers after the decimal to be in the thousandths place value location.)}$$


$$16/50 = 0.32$$

To change a fraction into a decimal fraction when the denominator is not 10, 100, or 1 000, you must create an equivalent fraction with a denominator of 10, 100 or 1 000.

$$\frac{1}{5} = \frac{2}{10} = 0.2 \quad \frac{20}{100} = 0.20 \quad \frac{200}{1\,000} = 0.200 \quad \left(\frac{1}{5} \text{ becomes } 0.2\right)$$

$$\frac{16}{50} = \frac{32}{100} = \frac{320}{1\,000}$$

$$\frac{32}{100} = 0.32, \quad \frac{320}{1\,000} = 0.320 \quad \left(\frac{16}{50} \text{ becomes } 0.32\right)$$

To change a mixed number into an improper fraction is a three-step process.

Step 1: Multiply the whole number by the denominator of the fraction.

Step 2: Add the numerator of the fraction to the answer in step 1.

Step 3: The answer in step 2 becomes the numerator of the improper fraction. The original denominator of the fraction is the denominator of the improper fraction.

Write $3\frac{4}{5}$ as an improper fraction:

Step 1: $3 \times 5 = 15$ (Multiply the whole number by the denominator.)

Step 2: $4 + 15 = 19$ (Add the numerator of the fraction to the answer in step 1 to find the numerator of the improper fraction.)

Step 3: The improper fraction is $\frac{19}{5}$ (The numerator is the answer in step 2 and the denominator of the improper fraction is the original denominator.)

To change an improper fraction to a mixed number, divide the numerator of the improper fraction by its denominator. Usually, your answer is a whole number and a remainder. The whole number becomes the first part of your mixed number and the remainder becomes the numerator of the fraction. The denominator does not change.

Change the following improper fractions to mixed numbers:

$$\frac{19}{6} = 3R1 \text{ (Divide 19 by 6 to get 3 remainder 1)}$$

The mixed number is $3 \frac{1}{6}$.

$$\frac{23}{5} = 4R3 \text{ (Divide 23 by 5 to get 4 remainder 3)}$$

The mixed number is $4 \frac{3}{5}$.

Questions

- Change the following fractions and mixed numbers into decimal fractions. The whole number in the mixed number is not changed.

Examples: $7 \frac{24}{100} = 7.24$ and $\frac{1}{10} = 0.1$

a. $\frac{3}{10} =$ _____ b. $\frac{24}{100} =$ _____

c. $\frac{334}{1\ 000} =$ _____ d. $4 \frac{8}{10} =$ _____

e. $29 \frac{567}{1\ 000} =$ _____ f. $12 \frac{87}{100} =$ _____

2. Change the following fractions and mixed numbers into decimal fractions. First, make an equivalent fraction that has a denominator that is 10, 100, or 1 000.

Examples: $2 \frac{7}{20} = 2 \frac{35}{100} = 2.35$ $2 \frac{7}{20}$ and 2.35 are equivalent

a. $\frac{1}{4} =$ _____

b. $\frac{4}{5} =$ _____

c. $6 \frac{3}{20} =$ _____

d. $\frac{10}{25} =$ _____

e. $8 \frac{40}{125} =$ _____

f. $13 \frac{13}{50} =$ _____



.75

3. Change the following fractions to their lowest-terms. Remember to make an equivalent fraction by **dividing**.

a. $\frac{6}{10} =$ (divide by 2) _____

b. $\frac{5}{20} =$ (divide by 5) _____

c. $\frac{15}{25} =$ (divide by 5) _____

d. $\frac{22}{50} =$ _____

e. $\frac{12}{24} =$ _____

f. $\frac{16}{92} =$ _____

4. Convert the following decimal numbers into fractions or mixed numbers. Then write the answer in the lowest terms.

Example: $1.8 = \frac{18}{10} = 1 \frac{4}{5}$ (8 and 10 are divisible by 2. The lowest terms fraction is $\frac{4}{5}$).

a. $0.6 =$ _____

b. $14.64 =$ _____

c. $0.75 =$ _____

d. $12.5 =$ _____

e. $2.25 =$ _____

f. $38.072 =$ _____

5. Write each of the following mixed numbers as improper fractions.

Examples: $2 \frac{1}{2} = \frac{5}{2}$ and $3 \frac{3}{4} = \frac{15}{4}$

a. $6 \frac{4}{7} =$ _____

b. $5 \frac{2}{3} =$ _____

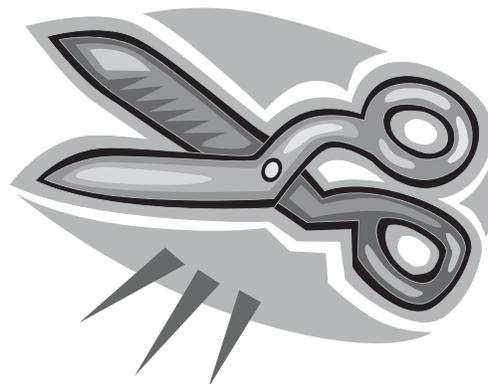
c. $9 \frac{1}{2} =$ _____

d. $3 \frac{7}{8} =$ _____

e. $12 \frac{2}{5} =$ _____

f. $15 \frac{2}{9} =$ _____

g. $20 \frac{3}{10} =$ _____



6. Write each of the following improper fractions as mixed numbers.

Examples: $\frac{11}{3} = 3R2$ The mixed number is $3 \frac{2}{3}$

$\frac{15}{2} = 7R1$ The mixed number is $7 \frac{1}{2}$

a. $\frac{9}{2} =$ _____

b. $\frac{13}{3} =$ _____

c. $\frac{22}{4} =$ _____

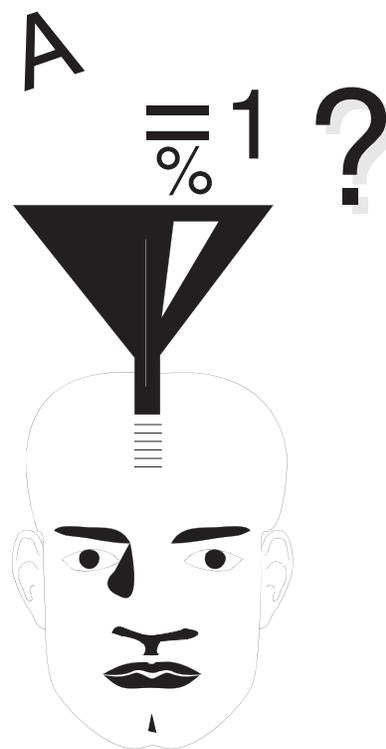
d. $\frac{19}{6} =$ _____

e. $\frac{27}{8} =$ _____

f. $\frac{37}{7} =$ _____

g. $\frac{49}{4} =$ _____

h. $\frac{55}{12} =$ _____



Comparing Proper Fractions, Improper Fractions, and Mixed Numbers

We describe fractions as having **common denominators** if the denominators in two or more fractions are the same.

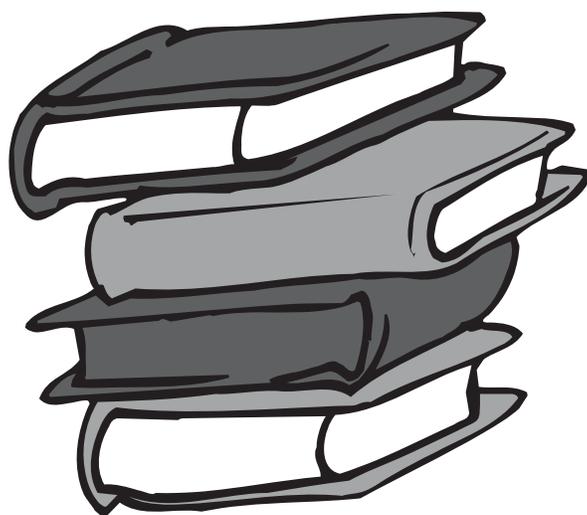
For example, the fractions $\frac{21}{25}$ and $\frac{17}{25}$ have common denominators.

When the denominators of two or more fractions are the same, you can easily tell which fraction is the largest by reading the numbers in the numerators.

Example: Arrange the following fractions in order according to size.

Put the largest fraction first, the fractions are $\frac{23}{100}$, $\frac{55}{100}$ and $\frac{3}{100}$.

Answer: $\frac{55}{100}$ (largest), $\frac{23}{100}$ (second largest), $\frac{3}{100}$ (smallest)



To arrange fractions according to size when the fractions have denominators that are not the same size, begin by creating equivalent fractions so that all the fractions have equal denominators.

Example: Arrange the following fractions according to size. Put the largest fractions first. Fractions are $\frac{3}{20}$, $\frac{6}{10}$, $\frac{2}{5}$ and $\frac{1}{4}$. All of the denominators are multiples of 20, so 20 will be the common denominator.

$\frac{3}{20} = \frac{3}{20}$ This fraction does not have to be changed.

$\frac{6}{10} = \frac{12}{20}$ In this fraction we can multiply the numerator and the denominator by 2 to get an equivalent fraction of $\frac{12}{20}$.

$\frac{2}{5} = \frac{8}{20}$ In this fraction we can multiply the numerator and the denominator by 4 to get an equivalent fraction of $\frac{8}{20}$.

$\frac{1}{4} = \frac{5}{20}$ In this fraction we can multiply the numerator and the denominator by 5 to get an equivalent fraction of $\frac{5}{20}$.

Answer: $\frac{6}{10} = \frac{12}{20}$ (largest), $\frac{2}{5} = \frac{8}{20}$ (second largest),

$\frac{1}{4} = \frac{5}{20}$ (third largest), $\frac{3}{20} = \frac{3}{20}$ (smallest).



Questions

1. Arrange the following fractions in order from greatest to least.

a. $\frac{12}{25}, \frac{24}{25}, \frac{2}{25}, \frac{18}{25}, \frac{33}{25}$

b. $\frac{56}{50}, \frac{72}{50}, \frac{2}{50}, \frac{43}{50}, \frac{59}{50}$

c. $\frac{5}{16}, \frac{9}{16}, \frac{15}{16}, \frac{7}{16}, \frac{25}{16}$

d. $\frac{66}{20}, \frac{33}{20}, \frac{19}{20}, \frac{6}{20}, \frac{64}{20}$

2. Arrange the following fractions in order from greatest to least. First, create equivalent fractions so that all the fractions have common denominators.

a. $\frac{3}{2}, \frac{8}{4}, \frac{3}{8}, \frac{4}{4}, \frac{5}{8}$ (Hint: Use 8 as the common denominator.)

b. $\frac{22}{50}, \frac{45}{100}, \frac{61}{100}, \frac{9}{50}, \frac{99}{100}$ (Hint: Use 100 as the common denominator.)

c. $\frac{11}{10}, \frac{11}{100}, \frac{11}{1\,000}, \frac{41}{100}$ (Hint: Use 1 000 as the common denominator.)

d. $\frac{2}{6}, \frac{5}{6}, \frac{3}{12}, \frac{11}{12}, \frac{17}{24}$ (Hint: Use 24 as the common denominator.)

3. Arrange the following fractions, decimals, and mixed numbers in order from greatest to least. Change all the mixed numbers and fractions into decimals, and then arrange them by size from largest to smallest.

Example: $6 \frac{1}{2}$, 8.45, $\frac{18}{18}$, 6.71, $8 \frac{2}{5}$

$$6 \frac{1}{2} = 6.5, \quad 8.45 = 8.45, \quad \frac{18}{18} = 1.0, \quad 6.71 = 6.71, \quad 8 \frac{2}{5} = 8 \frac{4}{10} = 8.4$$

First, change $8 \frac{2}{5}$ to $8 \frac{4}{10}$ (equivalent fraction), and then change the mixed number to a decimal. Answer: 8.45, 8.4, 6.71, 6.5, and 1.0.

a. $\frac{75}{100}$, 0.78, $\frac{75}{1\ 000}$, $\frac{7}{10}$, 0.65

b. $6 \frac{2}{10}$, $6 \frac{3}{20}$, $6 \frac{2}{5}$, $6 \frac{1}{10}$, $\frac{60}{10}$

c. 7.125, $72 \frac{5}{10}$, $7 \frac{1}{10}$, $712 \frac{5}{10}$, 71.25

4. Insert the correct sign to make the following statements true (> or < or =). Change each fraction and mixed number to a decimal and then compare the answers.

Example: Compare $\frac{12}{5}$ and $2\frac{1}{2}$.

$$\frac{12}{5} = 2R2 = 2\frac{2}{5} = 2\frac{4}{10} = 2.4$$

$$2\frac{1}{2} = 2\frac{5}{10} = 2.5$$

Answer: $2.4 < 2.5$ (2.4 is less than 2.5)

a. $\frac{3}{4}$ _____ 0.75

b. $\frac{12}{5}$ _____ $2\frac{2}{5}$

c. 1.88 _____ $1\frac{1}{2}$

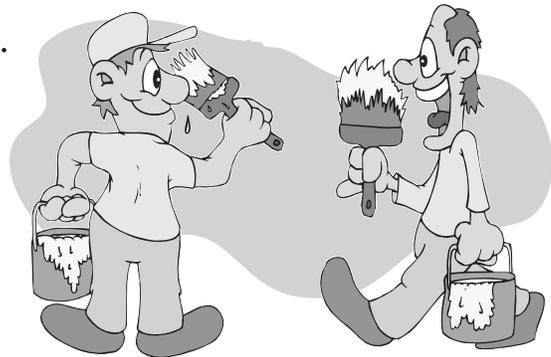
d. $4\frac{8}{9}$ _____ $\frac{44}{9}$

e. $15\frac{2}{5}$ _____ 15.5

f. $\frac{16}{10}$ _____ 1.66

5. Three students—Sue, Demetra, and Shere—painted a large mural on the end wall of the school gymnasium. Sue used $2\frac{1}{10}$ L of green and $\frac{3}{10}$ L of red paint. Demetra used $2\frac{7}{10}$ L of blue and $1\frac{1}{2}$ L of black paint. Shere used $\frac{4}{10}$ L of brown, $\frac{3}{10}$ L of yellow, $\frac{9}{10}$ L of purple, and $1\frac{1}{2}$ L of orange paint. Answer the following questions. Write your answers as fractions or mixed numbers.

Hint: $\frac{1}{2}$ L = $\frac{5}{10}$ L.



- How much paint did Sue use?
- How much paint did Demetra use?
- How much paint did Shere use?
- How much paint was used altogether?

6. On Monday, Billie celebrated her birthday with her friends. Each

student ate a fraction of the cake: Mary ate $\frac{1}{6}$,

Ron ate $\frac{1}{8}$, Carol ate $\frac{3}{12}$, Bob ate $\frac{5}{24}$, and

Kasey ate $\frac{1}{12}$. How much cake was left for Billie?



(Hints: This question has two parts. Use 24 as the common denominator.)

Homework Assignment

Use the following fractions and mixed numbers to answer the directions below:

$$\frac{3}{2}, \frac{4}{8}, \frac{15}{6}, \frac{5}{25}, 9\frac{7}{4}, 1\frac{3}{7}, \frac{3}{6}, 8\frac{6}{11}, \frac{5}{8}, \frac{10}{7}$$

1. Find and write three proper fractions.

2. Find and write three improper fractions.

3. Find and write three mixed numbers.

4. Find and write three fractions that are lowest-term fractions.

5. Find and write two fractions that have common denominators.

6. Find and write two fractions that are equivalent fractions.

7. Find and write the mixed number and the improper fraction that are equivalent.

Self-Evaluation

Ask yourself some important questions. Write your answers in sentences for your teacher.

1. In this lesson, what part of your work was **excellent**?

2. In this lesson, what part of your work **needs improvement**?

3. If you want help for some of the work in this lesson, ask your teacher in this space.
